

Octet magnetic moments and the Coleman-Glashow sum rule violation in the chiral quark model

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Baryon octet magnetic moments when calculated within the chiral quark model, incorporating the orbital angular momentum as well as the quark sea contribution through the Cheng-Li mechanism, not only show improvement over the nonrelativistic quark model results but also give a nonzero value for the right-hand side of the Coleman-Glashow sum rule. When effects due to spin-spin forces between constituent quarks as well as “mass adjustments” due to confinement are added, it leads to an excellent fit for the case of p, Σ^+, Ξ^0 and violation of the Coleman-Glashow sum rule, whereas in almost all the other cases the results are within 5% of the data.

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The European Muon Collaboration (EMC) measurements [1] in deep inelastic scattering have shown that only a small fraction of the proton’s spin is carried by the valence quarks. This “unexpected” conclusion from the point of view of the nonrelativistic quark model (NRQM), usually referred to as the “proton spin crisis,” becomes all the more intriguing when it is realized that the NRQM is able to give a reasonably good description of magnetic moments using the assumption that magnetic moments of quarks are proportional to the spin carried by them. This issue regarding spin and magnetic moments further becomes difficult to understand when it is realized that the magnetic moments of baryons receive a contribution not only from the magnetic moments carried by the valence quarks but also from various complicated effects, such as orbital excitations [2], sea quark polarization [3–6], effects of the chromodynamic spin-spin forces [7,8], effect of the confinement on quark masses [9], pion cloud contributions [10], loop corrections [11], relativistic and exchange current effects [12], etc. In the absence of any consistent way to calculate these effects simultaneously, it is very difficult to know their relative contributions. However, the success of NRQM, when viewed in this context, suggests that the various effects mentioned above contribute in a manner in which a large number of these are mutually canceled, making the understanding of the magnetic moments along with “spin crisis” all the more difficult. The problem regarding magnetic moments gets further complicated when one realizes that the Coleman-Glashow sum rule [13] (CGSR), valid in a large variety of models [14,15], is convincingly violated by the data [16]. For example, the CGSR for the baryon magnetic moments is given as

$$\Delta CG = \mu_p - \mu_n + \mu_{\Sigma^-} - \mu_{\Sigma^+} + \mu_{\Xi^0} - \mu_{\Xi^-} = 0. \quad (1)$$

Experimentally $\Delta CG = 0.49 \pm 0.05$ [16], clearly depicting the violation of CGSR by ten standard deviations. As $\Delta CG = 0$, in most of the calculations, obtaining $\Delta CG \neq 0$ along with the octet magnetic moments as well as resolution for “spin crisis” and related issues could perhaps provide vital clues for the dynamics as well as the appropriate degrees of freedom required for understanding some of the nonperturbative aspects of QCD.

In this context, it is interesting to note that the chiral quark model (χ QM) [4,17] with SU(3) symmetry is not only able to give a fair explanation of “proton spin crisis” [1] but is also able to give a fair account of $u-\bar{d}$ asymmetry [18–20] as well as the existence of significant strange quark content s in the nucleon [21]. Further, χ QM with SU(3) symmetry is also able to provide a fairly satisfactory explanation for baryon magnetic moments [3,4,17] as well as the absence of polarizations of the antiquark sea in the nucleon [22]. The predictions of the χ QM, particularly in regard to hyperon decay parameters [23], can be improved if symmetry breaking effects [5,6] are taken into account. However, χ QM with symmetry breaking, although gives a fairly good description of magnetic moments, is not able to describe ΔCG without resorting to additional parameters [15,24].

In a recent interesting work, Cheng and Li [25] showed that, within χ QM with SU(3) symmetry breaking, the long-standing puzzle why the NRQM is able to give a fair description of baryon magnetic moments can be resolved if one considers that the pions acting as Goldstone bosons (GBs) also have angular momentum and therefore contribute to the baryon magnetic moments as well. However, this contribution gets effectively canceled by the sea quark polarization effect, leaving the description of magnetic moments in terms of the valence quarks in accordance with the NRQM hypothesis. One can easily examine that the Cheng and Li proposal does not lead to an exact cancellation of the sea and orbital part for all the baryons, therefore its implications need to be examined in detail for the octet baryon magnetic moments. In particular, it would be interesting to explore the possibility of obtaining nonzero ΔCG by invoking the Cheng-Li mechanism along with the mass and coupling breaking effects.

To begin with, we consider the essentials of χ QM having bearings on the Cheng-Li mechanism. In χ QM, the basic process is the emission of a GB which further splits into a $q\bar{q}$ pair, for example,

$$q_{\pm} \rightarrow GB^0 + q'_{\mp} \rightarrow (q\bar{q}') + q'_{\mp}. \quad (2)$$

The effective Lagrangian describing interaction between quarks and the octet GBs and singlet η' is $\mathcal{L} = g_8 q \phi q$, where g_8 is the coupling constant and

$$\phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta \frac{\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} & \alpha K^0 \\ \alpha K^- & \alpha \bar{K}^0 & -\beta \frac{2\eta}{\sqrt{6}} + \zeta \frac{\eta'}{\sqrt{3}} \end{pmatrix}.$$

SU(3) symmetry breaking is introduced by considering different quark masses $m_s > m_{u,d}$ as well as by considering the masses of GBs to be nondegenerate ($M_{K,\eta} > M_\pi$) [5,6,15], whereas the axial U(1) breaking is introduced by $M_{\eta'} > M_{K,\eta}$ [4–6,15]. The parameter $a(=|g_8|^2)$ denotes the transition probability of chiral fluctuation of the splittings $u(d) \rightarrow d(u) + \pi^{+(-)}$, whereas $\alpha^2 a$, $\beta^2 a$, and $\zeta^2 a$ denote the probabilities of transitions of $u(d) \rightarrow s + K^{-(0)}$, $u(d,s) \rightarrow u(d,s) + \eta$, and $u(d,s) \rightarrow u(d,s) + \eta'$, respectively.

The magnetic moment corresponding to a given baryon can be written as

$$\mu_{\text{total}} = \mu_{\text{val}} + \mu_{\text{sea}} + \mu_{\text{orbit}}, \quad (3)$$

where $\mu_{\text{val}} = \sum_{q=u,d,s} \Delta q_{\text{val}} \mu_q$, $\mu_{\text{sea}} = \sum_{q=u,d,s} \Delta q_{\text{sea}} \mu_q$, μ_q ($q=u,d,s$) is the quark magnetic moment, and Δq ($q=u,d,s$) represents the net spin polarization and is defined as $\Delta q = q_+ - q_- + \bar{q}_+ - \bar{q}_-$ [5,6]. The valence contribution Δq_{val} can easily be calculated [4–6,15]. For example, for a proton we have $\Delta u_{\text{val}} = [\frac{4}{3}]$, $\Delta d_{\text{val}} = [-\frac{1}{3}]$ and $\Delta s_{\text{val}} = 0$. Similarly one can calculate for other baryons. The sea contribution in the χ QM basically comes from the splitting of GB into a $q\bar{q}$ pair [Eq. (2)]; its contribution to baryon magnetic moments can easily be calculated within χ QM [4,6,15]. For the case of a proton, it is given as

$$\begin{aligned} \Delta u_{\text{sea}} &= -\frac{a}{3} \left(7 + 4\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2 \right), \\ \Delta d_{\text{sea}} &= -\frac{a}{3} \left(2 - \alpha^2 - \frac{1}{3}\beta^2 - \frac{2}{3}\zeta^2 \right), \\ \Delta s_{\text{sea}} &= -a\alpha^2. \end{aligned} \quad (4)$$

Similarly, one can calculate Δq_{sea} and the consequent contribution to magnetic moments for other baryons. Following Cheng and Li [25], the μ_{orbit} for a χ QM can easily be evaluated and for a proton it is given as

$$\mu_{\text{orbit}} = \Delta u_{\text{val}} [\mu(u_+ \rightarrow)] + \Delta d_{\text{val}} [\mu(d_+ \rightarrow)], \quad (5)$$

where $\mu(u_+ \rightarrow)$ and $\mu(d_+ \rightarrow)$ are the orbital moments of u and d quarks and are given as

$$\begin{aligned} \mu(u_+ \rightarrow) &= \frac{a}{2M_{\text{GB}}(M_u + M_{\text{GB}})} \\ &\times [3(\alpha^2 + 1)M_u^2 + (\frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2 - \alpha^2)M_{\text{GB}}^2] \mu_N, \end{aligned} \quad (6)$$

$$\mu(d_+ \rightarrow) = \frac{a}{4M_{\text{GB}}(M_d + M_{\text{GB}})} \frac{M_u}{M_d} [(3 - 2\alpha^2 - \frac{1}{3}\beta^2 - \frac{2}{3}\zeta^2) \times M_{\text{GB}}^2 - 6M_d^2] \mu_N, \quad (7)$$

where M_q and M_{GB} are the masses of the quark and the GB, respectively, and μ_N is the Bohr magneton. In a similar manner, one can calculate the contributions for other baryons.

Before discussing the results, we would like to discuss the various inputs pertaining to χ QM, which include the mass and coupling breaking effects. The detailed analysis, incorporating the latest E866 data [19], to fit quantities such as $\bar{u}-\bar{d}$, $\bar{u}/\bar{d}$, Δu , Δd , Δs , G_A/G_V , Δ_8 , etc., will be discussed elsewhere, however here we would like to mention the values of the parameters used for the calculations of magnetic moments. For example, the pion fluctuation parameter a is taken to be 0.1, in accordance with most of the other calculations [3–6,15], whereas the coupling breaking parameters are found to be $\alpha=0.6$, $\beta=0.9$, and $\zeta=-0.3-\beta/2$. The orbital angular momentum contributions are characterized by the parameters of χ QM as well as the masses of the GBs, however the contributions are dominated by the pions, therefore the effects of other GBs have been ignored in the numerical calculations. For evaluating the contribution of pions, we have used its on mass shell value in accordance with several other similar calculations [26]. In the absence of any definite guidelines for the constituent quark masses, we have used the most widely accepted values in hadron spectroscopy [27–30], for example $M_u=M_d=330$ MeV, whereas the strange quark mass is taken from the NRQM sum rule $\Lambda - N = M_s - M_u$. These quark masses are then used to calculate μ_u , μ_d , and μ_s .

In Table I, we have presented the results of our calculations. From the table, one can easily find out that the χ QM with symmetry breaking is not only able to give $\Delta \text{CG} \neq 0$ but is also able to give a satisfactory description of data. When compared with NRQM results, interestingly one finds that the results either show improvement or they have the same overlap with the data as that of NRQM. Specifically, in p , Σ^- , and Ξ^0 one finds that there is a good deal of improvement as compared to NRQM results. In fact, from the table one can easily find out that except for Σ^- and Ξ^- , with a marginal correction with an opposite sign for the latter, in all other cases the sea+orbital contribution adds to the overall magnetic moments with the right sign. These conclusions remain largely valid when one considers variation in the coupling breaking parameters α and β . In particular, for the case of NMC data [18], with different values of α and β , we

TABLE I. Octet baryon magnetic moments in units of μ_N . The details of inputs are discussed in the text.

Octet baryons	Data [16]	χ QM					χ QM with mass adjustments				χ QM with mass adjustments and configuration mixing			
		NRQM	Valence	Sea	Orbital	Total	Valence	Sea	Orbital	Total	Valence	Sea	Orbital	Total
p	2.79	2.72	3.00	-0.70	0.54	2.84	3.17	-0.59	0.45	3.03	2.94	-0.55	0.41	2.80
n	-1.91	-1.81	-2.00	0.34	-0.41	-2.07	-2.11	0.24	-0.37	-2.24	-1.86	0.20	-0.33	-1.99
Σ^-	-1.16	-1.01	-1.12	0.13	-0.29	-1.28	-1.08	0.08	-0.26	-1.26	-1.05	0.07	-0.22	-1.20
Σ^+	2.45	2.61	2.88	-0.69	0.45	2.64	2.80	-0.55	0.37	2.62	2.59	-0.50	0.34	2.43
Ξ^0	-1.25	-1.41	-1.53	0.37	-0.23	-1.39	-1.53	0.22	-0.16	-1.47	-1.32	0.21	-0.13	-1.24
Ξ^-	-0.65	-0.50	-0.53	0.09	-0.06	-0.50	-0.59	0.06	-0.01	-0.54	-0.61	0.06	-0.01	-0.56
Λ	-0.61	-0.59	-0.65	0.10	-0.08	-0.63	-0.69	0.05	-0.04	-0.68	-0.59	0.04	-0.04	-0.59
Δ CG	0.49 ± 0.05	0				0.10				0.46				0.48

again find Δ CG $\neq 0$ as well as improvement over the NRQM results. The success of χ QM along with the Cheng-Li mechanism looks all the more impressive when one realizes that none of the magnetic moments have been used as inputs, suggesting that this mechanism seems to be providing the dominant dynamics of the constituent quarks and the GBs, the “appropriate” degrees of freedom of QCD in the nonperturbative regime.

After having seen that χ QM with Cheng-Li mechanism could provide the dominant dynamics of constituent quarks and the weakly interacting GBs, it would be natural to consider, as instances of further improvements, certain effects which can be easily incorporated in χ QM with the Cheng-Li mechanism. To this end, we have considered certain phenomenological effects among constituent quarks such as configuration mixing, known to be compatible with the χ QM [31,32], and the effects of “confinement,” both of which have been shown to improve the performance of NRQM [9,27–30]. For the present purpose, the effect of configuration mixing on the octet baryon wave function can be expressed as [8,27]

$$|8, \frac{1}{2}^+\rangle = \cos \phi |56, 0^+\rangle_{N=0} + \sin \phi |70, 0^+\rangle_{N=2}. \quad (8)$$

For details of the wave function we refer the reader to Refs. [7,8,27]. The angle ϕ can be fixed by consideration of the neutron charge radius [27]. This effectively leads to a change in the valence quark spin structure. For example, for a proton we have $\Delta u_{\text{val}} = \cos^2 \phi [\frac{4}{3}] + \sin^2 \phi [\frac{2}{3}]$, $\Delta d_{\text{val}} = \cos^2 \phi [-\frac{1}{3}] + \sin^2 \phi [\frac{1}{3}]$, and $\Delta s_{\text{val}} = 0$. These expressions would replace Δu_{val} and Δd_{val} in Eq. (5) for calculating the effects of configuration mixing on the orbital part. Again, for a proton, the sea quark contribution with configuration mixing can easily be calculated [33] and is expressed as

$$\Delta u_{\text{sea}} = -\cos^2 \phi \left[\frac{a}{3} (7 + 4\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2) \right] - \sin^2 \phi \left[\frac{a}{3} (5 + 2\alpha^2 + \frac{2}{3}\beta^2 + \frac{4}{3}\zeta^2) \right], \quad (9)$$

$$\Delta d_{\text{sea}} = -\cos^2 \phi \left[\frac{a}{3} (2 - \alpha^2 - \frac{1}{3}\beta^2 - \frac{2}{3}\zeta^2) \right] - \sin^2 \phi \left[\frac{a}{3} (4 + \alpha^2 + \frac{1}{3}\beta^2 + \frac{2}{3}\zeta^2) \right], \quad (10)$$

$$\Delta s_{\text{sea}} = -a\alpha^2. \quad (11)$$

One can perform similar calculations for other baryons; the details of these calculations will be presented elsewhere. Before including the effects of configuration mixing on magnetic moments, because of the changed valence quark spin distribution functions, one has to carry out a reanalysis to fit $\bar{u}-\bar{d}$, $\bar{u}/\bar{d}$, Δu , Δd , Δs , G_A/G_V , Δ_8 , etc., resulting in $\alpha = 0.4$, $\beta = 0.7$, and $\zeta = -0.3 - \beta/2$. Further, apart from taking $M_u = M_d = 330$ MeV, one has to consider the strange quark mass implied by the various sum rules derived from the spin-spin interactions for different baryons [8,27]. For example, $\Lambda - N = M_s - M_u$, $(\Sigma^* - \Sigma)/(\Delta - N) = M_u/M_s$, and $(\Xi^* - \Xi)/(\Delta - N) = M_u/M_s$, respectively, fix M_s for Λ , Σ , and Ξ baryons.

It has been shown that the effects of confinement can be simulated by “adjusting” the baryon masses [9], which leads to the following adjustments for the quark magnetic moments: $\mu_d = -[1 - (\Delta M/M_B)]\mu_N$, $\mu_s = -M_u/M_s[1 - (\Delta M/M_B)]\mu_N$, and $\mu_u = -2\mu_d$, where M_B is the mass of the baryon obtained additively from the quark masses and ΔM is the mass difference between the experimental value and M_B .

In Table I, we have included the results obtained after including the effects of configuration mixing and “mass adjustments.” In case of “mass adjustment,” one finds a remarkable improvement for Δ CG due to the difference in the valence contributions getting affected by the right magnitude, however the individual magnetic moments get disturbed. Interestingly, the situation changes completely when configuration mixing is also included. From the table, it is evident that we are able to get an excellent fit for almost all the baryons. It is almost perfect for p , Σ^+ , and Ξ^0 , in the case of n , Σ^- , and Λ , the value is reproduced within 5% of

experimental data. Only in the case of Ξ^- is the deviation somewhat more than 5%. Besides this, we have also been able to get an excellent fit to ΔCG . The fit becomes all the more impressive when it is realized that none of the magnetic moments are used as inputs. It may be of interest to mention that the fit in the case of Ξ^- can perhaps be improved in the present case if corrections due to pion loops are included [10]. In fact, a cursory look at Ref. [10] suggests that pion loop corrections would compensate Ξ^- much more compared to other baryons, hence providing an almost perfect fit.

To summarize our conclusions, we have carried out a detailed calculation of the octet magnetic moments in the χQM by including orbital and sea contributions through the Cheng-Li mechanism [25] with mass and coupling breaking effects. Apart from reproducing $\Delta\text{CG} \neq 0$, we are able to

show that Cheng-Li mechanism is able to improve the results of NRQM without any additional inputs, indicating that this mechanism provides the dominant dynamics of the constituent quarks and the Goldstone bosons. This is further borne out by the fact that when effects such as configuration mixing due to spin-spin interactions and “mass adjustments” due to confinement are added, we are able to get an excellent fit for the octet baryon magnetic moments as well as for ΔCG . It is almost perfect for p , Σ^+ , and Ξ^0 , whereas in the case of n , Σ^- , and Λ , the value is reproduced within 5% of experimental data.

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